**PPL – Assignment 4**

**Question 1**

1. False. If is of type , then the application of to will return an element of type , which will then be passed on to which expects an argument of type, resulting in an error. If is not of type then the application of to will throw an error.
2. True. Since is of type , then applying to it will indeed return an element of type .

**Question 2**

b) ((lambda (f1 x1) (f1 x1 1)) 4 +)

**Stage 1:** renaming

**Stage 2:** Assigning variables to each sub-expression

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| **Expression** | **Variable** |
| ((lambda (f x) (f x 1)) 4 +) | T0 |
| (lambda (f x) (f x 1)) | T1 |
| (f x 1) | T2 |
| f | Tf |
| x | Tx |
| 4 | Tnum4 |
| + | T+ |

**Stage 3:** Construct type equations

The equations for the type expressions are

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| **Expression** | **Equation** |
| ((lambda (f x) (f x 1)) 4 +) | T1 = [Tnum4 \* T+ -> T0] |
| (lambda (f x) (f x 1)) | T1 = [Tf \* Tx -> T2] |
| (f x 1) | Tf = [Tx -> T2] |

The equations for the primitives are

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| **Expression** | **Equation** |
| 4 | Tnum4 = Number |
| + | T+ = [Number \* Number -> Number] |

**Stage 4:** Solve the equations.

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| **Equation** | **Substitution** |
| 1. T1 = [Tnum4 \* T+ -> T0] | {} |
| 1. T1 = [Tf \* Tx -> T2] |  |
| 1. Tf = [Tx -> T2] |  |
| 1. Tnum4 = Number |  |
| 1. T+ = [Number \* Number -> Number] |  |

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| **Equation** | **Substitution** |
| 1. T1 = [Tf \* Tx -> T2] | {T1 := [Tnum4 \* T+ -> T0]} |
| 1. Tf = [Tx -> T2] |  |
| 1. Tnum4 = Number |  |
| 1. T+ = [Number \* Number -> Number] |  |

T1 = [Tf \* Tx -> T2] ○ Substitution = ([Tnum4 \* T+ -> T0] = [Tf \* Tx -> T2]) There is not type-sub since both sides of the equation are composite, we split it into three equations (6,7,8) and remove equation 2.

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| **Equation** | **Substitution** |
| 1. Tf = [Tx -> T2] | {T1 := [Tnum4 \* T+ -> T0]} |
| 1. Tnum4 = Number |  |
| 1. T+ = [Number \* Number -> Number] |  |
| 1. Tf = Tnum4 |  |
| 1. Tx = T+ |  |
| 1. T2 = T0 |  |

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| --- | --- |
| **Equation** | **Substitution** |
| 1. Tnum4 = Number | {T1 := [Tnum4 \* T+ -> T0], Tf = [Tx -> T2]} |
| 1. T+ = [Number \* Number -> Number] |  |
| 1. Tf = Tnum4 |  |
| 1. Tx = T+ |  |
| 1. T2 = T0 |  |

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| **Equation** | **Substitution** |
| 1. T+ = [Number \* Number -> Number] | {T1 := [Number \* T+ -> T0], Tf = [Tx -> T2], Tnum4 = Number } |
| 1. Tf = Tnum4 |  |
| 1. Tx = T+ |  |
| 1. T2 = T0 |  |

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| **Equation** | **Substitution** |
| 1. Tf = Tnum4 | {T1 := [Number \* [Number \* Number -> Number] -> T0], Tf = [Tx -> T2], Tnum4 = Number, T+ = [Number \* Number -> Number] } |
| 1. Tx = T+ |  |
| 1. T2 = T0 |  |

(Tf = Tnum4) ○ Substitution = ([Tx -> T2]=Number). We get incompatible types, cannot continue.